PROTON-NEUTRON OCCUPANCIES OF SPHERICAL SHELLS IN THE GROUND STATES OF SOME DOUBLY EVEN 2S-1D SHELL NUCLEI

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CERTIFICATE

This is to certify that the work, entitled ' Proton-Neutron Occupancies of Spherical Shells in the Ground States of some Doubly Even Nuclei' has been carried out by Mr. Malay Kumar Nandy under my supervision.

To the best of my knowledge this work is original and has not formed the basis for the award of any other degree.

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ABSTRACT

The aim of the project discussed in the thesis is twofold. First we assess the relative efficacy of the Projected Hartree-Fock-Bogoliubov (PHFB) approach for nuclear many-body problem. This is done by comparing the subshell occupation numbers resulting from the PHFB as well as the exact (configuration mixing) shell model approaches. Second, we discuss the calculation of subshell occupation numbers for a number of 2s-1d shell nuclei employing various effective interactions in conjunction with the PHFB method. The task is facilitated by the fact that the PHFB approach is considerably faster than the usual (shell model) configuration mixing method. A comparison of the calculated results with the available experimental information yields significant information regarding the suitability of various interactions for detailed microscopic studies in the 2s-1d shell.

CHAPTER 1

INTRODUCTION

The expectation value of the subshell occupancy operator is one of the simplest measurable dynamical quantities in the ground state of a nucleus. The measured values of the subshell occupation numbers provide a direct test of the nuclear wave functions involved in the microscopic description of the ground state.

A calculation of the subshell occupation numbers in terms of the exact shell model wave functions is not feasible for the 2s-1d shell nuclei; the dimensionalities of the Hamiltonian matrices involved are prohibitively large. Furthermore, the collective nature of the observed properties of the nuclei with $16<A<40$ necessitates the involvement of the full 2s-1d valence space, and the truncation of the shell model space is not expected to give accurate results.

Potbhare and Pandya[1] sometime ago reported the calculation of the ground state energies and the subshell occupation numbers of several 2s-1d shell nuclei in the framework of the spectral distribution method developed by French and co-workers.[2,3] The occupation numbers resulting from a number of effective interactions in the 2s-1d shell were compared. However, it turns out that the ground states

in various 2s-1d shell nuclei are usually more than five widths (σ) away from the centroid of the energy distributions. This severely constrains the usefulness of the spectral distribution method involving just the first two moments of the Hamiltonian; the results obtained via the spectral methods become very sensitive towards the higher-order shape correction to the configuration level density.

In contrast to the spectral methods, the Hartree-Fock (HF) or the Hartree-Fock-Bogoliubov (HFB) approaches[4] are a priori more suited for the calculation of subshell occupation numbers for the ground state resulting from a given effective interaction. The requirement of the minimization of energy for the HF or HFB ansatz of the variational wave function is connected primarily with the extraction of the one-body field. The lowest-energy HF/HFB intrinsic state, therefore, corresponds to the optimum single-particle field (resulting from two-body interactions) appropriate for the ground state. This single-particle field in turn provides the best set of (single-particle) wave functions for calculating the expectation values of various one-body operators in the ground states.

The aim of this paper is twofold. First we compare the subshell occupation numbers resulting from the projected Hartree-Fock-Bogoliubov (PHFB) and the shell-model approach

employing latest 2s-1d shell interactions; the comparison permits an assessment of the relative efficacy of the PHFB approach. Second, we report on the calculation of subshell occupation numbers for a number of 2s-1d shell nuclei employing the angular momentum projected HFB wave functions resulting from various effective interactions. A comparison of the calculated results with the available experimental ones provides information regarding the suitability of the effective interaction vis-à-vis a detailed microscopic study of various isotopes.

In Chapter 2 we give an outline of the formalism for the calculation of the proton and neutron subshell occupation numbers using variational HFB wave functions. Chapter 3 contains discussions on the theoretical results and their comparison with the available data obtained from various pickup and stripping reactions.

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CHAPTER 2

THEORETICAL FRAMEWORK

2-1 The Shell Model and the Residual Interaction

Normally a nucleus is a many body system, so we are concerned with solving the many body Schrodinger equation

$$H\Phi = E\Phi \tag{1}$$

where Φ is the intrinsic state of the nucleous assumed to be given by the Slater determinant:

$$\Phi \equiv \psi_{\beta_1\beta_2\ldots\beta_A}(x_1, x_2, \ldots x_A)$$

$$= \frac{1}{\sqrt{A!}} \begin{vmatrix} \phi_{\beta_1}(x_1) & \phi_{\beta_1}(x_2) & \ldots & \phi_{\beta_1}(x_A) \\ \phi_{\beta_2}(x_1) & \phi_{\beta_2}(x_2) & \ldots & \phi_{\beta_2}(x_A) \\ \ldots & \ldots & & \ldots \\ \phi_{\beta_A}(x_1) & \phi_{\beta_A}(x_2) & \ldots & \phi_{\beta_A}(x_A) \end{vmatrix} \tag{2}$$

$\phi_{\beta_i}(x_j)$ being the single-nucleon wave functions of the orbit β_i. The coordinates x_j include description of position, spin and other coordinates the wave functions may be dependent on.

The many-body nuclear Hamiltonian in eq.(1) is given by

$$H = \sum_{i=1}^{A} t_i + \sum_{i<j}^{A} V_{ij} \tag{3}$$

where A = No. of nucleons present in the nucleus

t_i = K.E. of the i^{th} nucleon

V_{ij} = $V(x_i, x_j)$ = The actual two nucleon interaction potential

The two-nucleon interaction is very strong and contains a short-range strongly repulsive core in some two-body states. So, its form is too much complicated and yet the exact form is unknown. Thus it is hardly possible to have an exact solution of eq. (1).

The success of the independent particle model in explaining a large number of experimental data suggests that the short-range rupulsive core does not play an important role in many nuclear properties. It is therefore believed that the wave functions which are the solutions of eq. (1) with an effective nucleon-nucleon interaction from which the repulsive core has been removed are significant.

In an attempt to solve the many body eq. (1) approximately, we assume that each nucleon of the nucleus moves independently in a potential field that is average of the interactions of this nucleon with all the other nucleons of the nucleus. Denoting this one-body interaction by $v_i = v(x_i)$ we have

$$H = \sum_{i=1}^{A} t_i + \sum_{i=1}^{A} v_i \qquad (4)$$

But in writing so we are neglecting a term, which is obvious when (3) is written as

$$H = \sum_{i=1}^{A} t_i + \sum_{i=1}^{A} v_i + \left(\sum_{i<j}^{A} V_{ij} - \sum_{i=1}^{A} v_i \right) \qquad (5)$$

We write the quantity in parentheses as

$$\left(\sum_{i<j}^{A} V_{ij} - \sum_{i=1}^{A} v_i \right) = \sum_{i<j}^{A} V_{ij}^{res}$$

where V_{ij}^{res} is the <u>residual</u> two-nucleon interaction that remains after the extraction of the one-body interaction from the effective two-nucleon interaction.

The correctness of the mathematical results of (4) depends upon how we choose the form of v_i.

In shell-model calculations we choose the form of v_i as

$$v_i = \frac{1}{2} M \omega^2 \vec{r}_i^{\,2} + \alpha \vec{l}_i \cdot \vec{s}_i + \gamma \vec{l}_i \cdot \vec{l}_i \qquad (6)$$

and set $V_{ij}^{res} = 0$ for all i,j. The oscillator parameter (ω) is chosen to match the observed r.m.s. radius of the nucleus and the strength of the spin-orbit coupling ($\bar{\alpha}, \bar{\gamma}$) are chosen to match the spin-orbit splitting.

The choice (6) permits one to make a number of useful prediction for core + one-particle systems. For nuclei involving a few valence particles one can attempt to diagonalize V^{res} <u>exactly</u> in the basis provided by (6) .

However,for larger (n>3) number of valence particles this procedure becomes quite cumbersome; the dimensionalities of the relevent Hamiltonian matrices become prohibitively large. In such cases one invokes the variational methods such as the HF or the HFB methods.

2-2 The Hartree-Fock Method[1,2]

In the Hartree-Fock formalism we denote the correct single-nucleon states $|\beta\rangle$ be expanded in terms of the basis states $|i\rangle$

$$|\beta\rangle = \sum_i C_i^{\beta} |i\rangle \; ; \quad 1 \leq \beta \leq A \tag{7}$$

The basis states may be taken to be the harmonic oscillator states (in the presence of spin-orbit coupling) used in ordinary shell-model calculations.

2-2-1 The Choice of an Expansion for the HF-orbits $|\beta\rangle$

The basis $|i\rangle$ on which HF-orbits $|\beta\rangle$ are expanded is arbitrary in principle. In practice, useful expansions are suggested by

i) the convergence of the expansion,

ii) the convenience of calculating the matrix elements of the two-body interaction, and

iii) the symmetries of the HF- field

Harmonic oscillator orbits were found to give excellent convergence for the expansion of bound orbits of spherical

closed-shell nuclei. Thus harmonic-oscillator wave functions can be used for the radial part of the basis state $|i\rangle$.

Shell-model states $|nljmT\rangle$ are convenient to use because of the general availability of matrix elements of the two-body interaction in this representation.

Expansion (7) for the HF-orbits on the shell-model basis is

$$|\beta\rangle = \sum_{nljm} C^{\beta}_{nljm} |nljmT_{\beta}\rangle \qquad (8)$$

Summation over T is avoided to ensure that proton and neutron states do not mix together; T is the third component of iso-spin $\vec{t}$, distinguishes proton and neutron states.

2-2-2 Symmetries of the HF-Field

Axial Symmetry:

The HF-orbits are said to be axially symmetric if each orbit $|\beta\rangle$ has a good projection of J_z i.e. the orbits are eigen states of J_z

$$J_z |\beta\rangle = m_{\beta} |\beta\rangle$$

Thus each state $|\beta\rangle$ is characterised by a given projection m_{β} and hence the expansion of an orbit β is limited to basis states of a given m_{β}

$$|\beta\rangle = \sum_{nlj} C^{\beta}_{nlj} |nljm_{\beta}T_{\beta}\rangle \qquad (9)$$

Obviously such orbits would correspond to a potential which has rotational symmetry about z-axis. Also the total wave function ϕ given by (2) will have a definite projection K

$$J_z|\phi_K\rangle = K|\phi_K\rangle$$

where $$K = m_{\beta_1} + m_{\beta_2} + \cdots + m_{\beta_A}$$

Time-Reversal Symmetry:

With real C^{β}_{nlj} coefficients, time reversal symmetry is equivalent to the symmetry of a rotation of π about the y - axis (equivalent to a rotation in the xz-plane) times the parity operation

$$T = e^{-i\pi J_y} P$$

In even-even nuclei, time-reversal symmetry is obtained by requiring that, for each occupied orbit

$$|\beta\rangle = \sum_{nljm} C^{\beta}_{nljm} |nljmT_{\beta}\rangle$$

the time-reversed orbit

$$|\bar{\beta}\rangle = T|\beta\rangle = \sum_{nljm} C^{\beta *}_{nljm} (-1)^{l+j-m} |nlj,-mT_{\beta}\rangle$$

is also occupied and orthogonal to $|\beta\rangle$. Then the HF-field commutes with the time-reversal operator and the orbits $|\beta\rangle$ and $|\bar{\beta}\rangle$ are degenerate. Also

$$T|\phi\rangle = |\phi\rangle$$

Obviously, in axially symmetric solutions the orbits $|\beta\rangle$ and $|\bar{\beta}\rangle$ are always orthogonal. In other solutions they are orthogonal if all the components of the orbits have either

$m+l+1/2$ even

or $m+l+1/2$ odd

i.e. expansion of each orbit is limited to basis states with either $m+l+1/2$ even or $m+l+1/2$ odd.

2-2-3 The Hartree-Fock Equations

We consider axial-symmetry so that each orbit $|\beta\rangle$ has a definite projection m_β and is expanded as

$$|\beta\rangle = \sum_j C_\beta^j \, |jm_\beta\rangle \qquad (10)$$

where j stands for nlj and m_β for $m_\beta T_\beta$. This expansion is subject to the normalization requirement

$$\sum_j |C_\beta^j|^2 = 1; \quad \text{for all} \quad \beta$$

The total nuclear state, given by the determinant in eq.(2) then can be labelled by the total projection K

i.e. $$|\phi_K\rangle = \frac{1}{\sqrt{A!}} \det [|\beta_1\rangle \, |\beta_2\rangle \ldots |\beta_A\rangle]$$

The energy of the system is obtained by taking the expectation value of the Hamiltonian given by (3) in the state $|\phi_K\rangle$:

$$E_{HF} = \langle\phi_K|H|\phi_K\rangle$$

$$= \sum_{\beta=1}^{A} \langle\beta|t|\beta\rangle + \sum_{\beta<\mu}^{A} \langle\beta\mu|V|\beta\mu\rangle$$

where E_{HF} is called the HF-energy of the system, $\langle\beta|t|\beta\rangle$ is the K.E. of a nucleon in the orbit β and $\langle\beta\mu|V|\beta\mu\rangle$ is an antisymmetrised matrix element of the two-body interaction V_{ij}.

On expanding the orbits $|\beta\rangle$ as given by (10) we have

$$E_{HF} = \langle \phi_K|H|\phi_K\rangle$$

$$= \sum_{\beta} \sum_{j_1 j_3} C_{\beta}^{j_1 *} C_{\beta}^{j_3} \langle j_1 m_\beta|t|j_3 m_\beta\rangle$$

$$+ \sum_{\beta<\mu} \sum_{j_1 j_3} C_{\beta}^{j_1 *} C_{\beta}^{j_3} \langle j_1 m_\beta;\mu|V|\, j_3 m_\beta;\, \mu\rangle \quad (11)$$

Now, the purpose of the HF theory is to determine the single-particle orbits $|\beta\rangle$ (eq.10) in the Slater determinant of $|\phi_K\rangle$. This is equivalent to the problem of determining the C_β^j coefficients in eq (10), since $|jm_\beta\rangle$ are known basis states (chosen to be shell-model states).

This is achieved by requiring the energy of the system (i.e. E_{HF}) stationary and minimum for infinitesimal variations of the urbits β (brought about by infinitesimal variations of the C_β^j coefficients) subject to the constraint given by eq (10). Thus we require

$$\frac{\partial}{\partial C_\beta^{j_1 *}} \left[\langle\phi_K|H|\phi_K\rangle - \sum_{\beta} \epsilon_\beta \sum_{j_1} C_\beta^{j_1 *} C_\beta^{j_1}\right] = 0 \quad (12)$$

where ϵ_β are introduced as Lagrange multipliers. With the help of the expansion given in eq (11), this derivative can be calculated directly:

$$\sum_{j_3} C_\beta^{j_3} \langle j_1 m_\beta | t | j_3 m_\beta \rangle + \sum_{\mu} \sum_{j_3} C_\beta^{j_3} \langle j_1 m_\beta ; \mu | V | j_3 m_\beta ; \mu \rangle = \epsilon_\beta C_\beta^{j_1}$$

which is the result after removing the common summations over β. This can also be written as:

$$\sum_{j_3} \left[\langle j_1 m_\beta | t | j_3 m_\beta \rangle + \sum_{\mu=1}^{A} \langle j_1 m_\beta ; \mu | V | j_3 m_\beta ; \mu \rangle \right] C_\beta^{j_3} = \epsilon_\beta \; C_\beta^{j_1} \qquad (13)$$

which has the form of an eigen value equation

$$\sum_{j_3} \langle j_1 m_\beta | h | j_3 m_\beta \rangle \; C_\beta^{j_3} = \epsilon_\beta \; C_\beta^{j_1} \qquad (14)$$

where

$$\langle j_1 m_\beta | h | j_3 m_\beta \rangle = \left[\langle j_1 m_\beta | t | j_3 m_\beta \rangle + \sum_{\mu=1}^{A} \langle j_1 m_\beta ; \mu | V | j_3 m_\beta ; \mu \rangle \right] \qquad (15)$$

h is called the HF hamiltonian for a single nucleon and the quantities ϵ_β are the HF single-particle energies. The states $|\beta\rangle$ are the HF single particle states and can be known on determining the C_β^j coefficients.

The equations (14) and (15) are the HF equations for the orbit β. They are solved by iteration procedure.

2-2-4 Expansion for the Occupied Orbits in the HF Equations

Under axial symmetry, the μ-orbits in eq (15) can be expanded as

$$|\mu\rangle = \sum_{j} C_{\mu}^{j} |jm_{\mu}\rangle$$

whence the two-body term in eq. (15) becomes

$$\sum_{\mu} \sum_{j_2} \sum_{j_4} C_{\mu}^{j_2 *} C_{\mu}^{j_4} \langle j_1 m_{\beta};\ j_2 m_{\beta} | V | j_3 m_{\beta};\ j_4 m_{\mu} \rangle$$

$$\neq \sum_{j_2} \sum_{j_4} \left(\sum_{\mu} C_{\mu}^{j_2 *} C_{\mu}^{j_4} \right) \langle j_1 m_{\beta};\ j_2 m_{\mu} | V | j_3 m_{\mu};\ j_4 m_{\mu} \rangle$$

Thus we can not separate out the factor

$$\rho_{j_2 j_4} = \sum_{\mu} C_{\mu}^{j_2 *} C_{\mu}^{j_4}$$

and hence can not define the density matrix $\hat{\rho}$. However, if we violate axial symmetry in the expansion of $|\mu\rangle$ i.e.

$$|\mu\rangle = \sum_{jm} C_{\mu}^{jm} |jm\rangle$$

we can define density matrix as shown below:

$$\sum_{\mu} \langle j_1 m_{\beta};\ \mu | V |\ j_3 m_{\beta};\ \mu \rangle$$

$$= \sum_{j_2 m} \sum_{j_4 m'} \left(\sum_{\mu} C_{\mu}^{j_2 m *} C_{\mu}^{j_4 m'} \right) \langle j_1 m_{\beta}; j_2 m | V | j_3 m_{\beta}; j_4 m' \rangle \qquad (16)$$

as we can separate out the factor in parantheses. They are matrix elements of the density operator $\hat{\rho}$:

$$\rho_{j_2 m; j_4 m'} = \sum_{\mu=1}^{A-16} C_{\mu}^{j_2 m *} C_{\mu}^{j_4 m'}$$

where jm stands for nljmT.

The uncoupled two-body interaction matrix elements in (16) are given in terms of antisymmetrized coupled two-body matrix elements by

$$\langle j_1 m_\beta T_\beta;\ j_2 m T | V |\ j_3 m_\beta T_\beta;\ j_4' m' T' \rangle$$

$$= \sum_{JK} \sum_{TM_T} \begin{bmatrix} j_1 & j_2 & J \\ m_\beta & m & K \end{bmatrix} \begin{bmatrix} j_3 & j_4 & J \\ m_\beta & m' & K \end{bmatrix} \begin{bmatrix} 1/2 & 1/2 & T \\ T_p & T & M_T \end{bmatrix} \begin{bmatrix} 1/2 & 1/2 & T \\ T_p & T' & M_T \end{bmatrix}$$

$$\times \langle (j_1 j_2) JT | V | (j_3 j_4) JT \rangle \qquad (17)$$

The quantities in square brackets are the Clebsch-Gordon coefficients:

$$\begin{bmatrix} j_1 & j_2 & J \\ m_1 & m_2 & M \end{bmatrix} = \langle j_1 m_1;\ j_2 m_2 | (j_1 j_2) JM \rangle$$

which are non zero only when $m_1+m_2=M$ and hence suggest that

$$m = m'$$
$$T = T'$$

2-2-5 Single Major Shell Hartree-Fock Calculations

In single major shell HF calculations the expansion (8) of each HF orbit is limited to states $|nljmT\rangle$ belonging to a major shell of the spherical oscillator. Thus for $(1d_{5/2}\ 2s_{1/2}\ 1d_{3/2})$ major shell calculation we expand the HF orbits as

$$|\beta\rangle = \sum_{nljm \in (1d,2s)} C^{\beta}_{nljm}\ |nljmT_\beta\rangle$$

for the valence nucleons in the (1d,2s) major shell. The inner shells which are completely filled are assumed to form an 'inert core', and are ignored in actual calculations. Also A should be replaced by (A-No. of nucleons in the core) = (A-16).

2-2-6 The use of a Reference Closed-Shell Nucleus

For 2s-1d shell nuclei we assume an inert ^{16}O closed-shell core plus A-16 valance nucleons occupying the orbits β which are expanded on the $1d_{5/2}, 2s_{1/2}, 1d_{3/2}$ states. Thus it is convenient to choose ^{16}O as a reference nucleus. Then the sum over μ in eq. (15) can be splitted as:

$$\sum_{\mu=1}^{A} = \sum_{\mu=1s,1p} + \sum_{\mu=1}^{A-16}$$

where the first summation is over therefore 1s-shell orbits and the twelve 2P-shell orbits of ^{16}O. eq.(15) then becomes

$$\langle j_1 m_\beta | h | j_3 m_\beta \rangle = [\langle j_1 m_\beta | t | j_3 m_\beta \rangle + \sum_{\mu=1s,1p} \langle j_1 m_\beta ; \mu | V | j_3 m_\beta ; \mu \rangle + \sum_{\mu=1}^{A-16} \langle J_1 m_\beta ; \mu | V | j_3 m_\beta ; \mu \rangle$$

The quantity in parantheses is the HF Hamiltonian of the closed shell ^{16}O. Thus the HF Hamiltonian takes the simple form

$$\langle j_1 m_\beta | h | j_3 m_\beta = e_{j_1} \delta_{j_1 j_3} + \sum_{\mu=1}^{A-16} \langle j_1 m_\beta ; \mu | V | j_3 m_\beta ; \mu \rangle \qquad (18)$$

where the K.E. term in (15) is replaced by the single-particle energies e_j of the ^{16}O field. These single-particle energies may be obtained from the binding-energy differences.

$$e_j = [^{17}O]_j - [^{16}O] \; ; \quad j \in (2s,1d)$$

where $[^{16}O]$ is the binding-energy of ^{16}O, and $[^{17}O]_j$ is the binding energy of a state of ^{17}O with spin j. Several other methods are there for the maipulation of e_j-values.

An important point is to note that, the e_j-values in (18) are the ones obtained with the ^{16}O value of $\alpha = \sqrt{M\omega/\hbar}$, the oscillator constant. As α varies from one nucleus to another, the e_j's also vary. This effect may be taken into account by taking parametrized values of e_j's which are available.

2-2-7 The Iterative Procedure

Finally we have the HF equations, for the 2s-1d major shell calculations,

$$1. \quad \sum_{j_3} \langle j_1 m_\beta \,|h|\, j_3 m_\rho \rangle C C_\beta^{j_3} = \epsilon_\rho \; C_\beta^{j_1} \tag{19}$$

$$2. \quad \langle j_1 m_\beta \,|h|\, j_2 m_\rho \rangle = e_{j_1} \delta_{j_1 j_3} + \sum_{\mu=1}^{A-16} \langle j_1 m_\rho ; \mu | V | j_3 m_\beta ; \mu \rangle$$

$$= e_{j_1} \delta_{j_1 j_3} + \sum_{j_2 m} \sum_{j_4 m} \left(\sum_{\mu=1}^{A-16} C_\mu^{j_2 m *} C_\mu^{j_4 m} \right)$$

$$\langle j_1 m_\rho ; \, j_2 m | V | j_3 m_\beta ; \, j_4 m \rangle \tag{20}$$

for the orbits β. They can be solved by <u>iteration</u> as follows:

a. An initial guess for the set of coefficients C_{β}^{jm} is made.

b. With this set of coefficients C_{β}^{jm}, the matrix elements of the HF Hamiltonian are calculated from eq (20).

c. Knowing the HF Hamiltonian matrix in the step b., it is diagonalised to solve the eigen-value eq (19). ϵ_{β}'s and a new set of coefficients C_{β}^{jm} are obtained.

Now we return to step b. This process is repeated until successive diagonalizations produce an almost equal set of coefficients C_{β}^{jm} (within desired degree of accuracy).

2-3 The Hartree-Fock-Bogoliubov method[1,3]

We consider the HF field possessing time-reversal symmetry. For even-even nuclei this symmetry is easily ensured by requiring that for each occupied orbit $|k>$, the time-reversed orbit $|\bar{k}>$ is also occupied.

The axially symmetric intrinsic deformed HFB state with K=0 can be written, according to the BCS theory of superconductivity, as

$$|\phi_{K=0}> = \prod_k (U_k + V_k\, b_k^+\, b_{\bar{k}}^+)\,|0> \tag{21}$$

This amounts to putting a pair in the states k and $\bar{k}$ with probability V_k^2 (Here V_k and U_k indicate respectively the fullness and emptiness of the orbit k). Thus $|\phi_o>$ contains a superposition of many pair-excited configurations each having different number of nucleons. In the case of the axially symmetric case, the creation operators can be expressed as

$$\begin{aligned} b_k^+ &= \sum_j C_k^j\; a_{jk}^+ \\ b_{\bar{k}}^+ &= \sum_j (-1)^{j-k}\, C_k^j\; a_{j-k}^+ \end{aligned} \tag{22}$$

where a_{jk}^+ creates a particle in the state $|jk>$.

The normalization of the state $|\phi_o>$ requires

$$U_k^2 + V_k^2 = 1 \tag{23}$$

In the HF limit $V_k = 1$, $U_k = 0$ for the occupied orbits and $V_k = 0$, $U_k = 1$ for the unoccupied orbits. Thus

$$|\phi_o^{HF}\rangle = \prod_{k(\text{occupied})} b_k^+ \; b_{\bar{k}}^+ \; |C\rangle \qquad (24)$$

We can perform a canonical transformation from the operators b_k^+ to a new set of operators q_k such that $|\phi_o\rangle$ represents a <u>vacuum</u> of the new operators q_k

$$q_k = U_k \; b_k - V_k \; b_{\bar{k}}^+$$

$$q_{\bar{k}} = U_k \; b_{\bar{k}} + V_k \; b_k^+ \qquad (25)$$

with $\quad q_k \; |\phi_o\rangle = 0$, for all k.

The Hamiltonian of the many-body system consisting of the non-interacting HF particles is given by

$$H^{(HF)} = \sum_k E_k^o \; b_k^+ \; b_k \qquad (26)$$

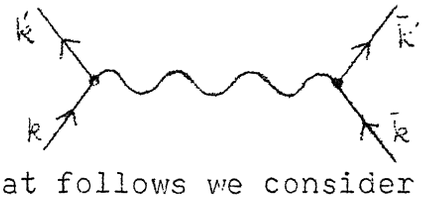
where E_k^o is the HF energy of the level k.

Though the HF theory extracts the <u>best</u> single-particle field from the residual two-body interactions, some parts of the latter are still left untreated. Therefore we invoke the '<u>pairing</u> ' interaction between the particles in the HF scheme; this interaction generates the admixtures of pair-excited configurations in the intrinsic wave function ϕ_o . We write the Hamiltonian as

$$H = \sum_k E_k^o \, b_k^+ \, b_k - \sum_{kk'} \langle \bar{k}' | \, V^{res} \, | k\bar{k} \rangle \, b_{k'}^+ b_{\bar{k}}^\pm, \, b_{\bar{k}} b_k \qquad (27)$$

The matrix elements of the pairing interaction

$G_{k'k} = \langle k'\bar{k}' | V^{res} | k\bar{k} \rangle$ are represented by the diagram



In what follows we consider the state-independent pairing interaction characterized by the average of the matrix elements $G_{k'k}$

$$G = \overline{\langle k'\bar{k}' | V^{res} | k\bar{k} \rangle} \qquad (28)$$

We can thus write eq. (27) as

$$H = \sum_k E_k^o \, b_k^+ \, b_k - G \sum_{kk'} b_{k'}^+, \, b_{\bar{k}}^\pm, \, b_{\bar{k}} \, b_k \qquad (29)$$

For G=0, this Hamiltonian goes over to the HF limit. As we can easily see, for $G \neq 0$ the pairing interactions smooth out the jumps in the occupation numbers for the HF orbits. Now the number operator is defined as

$$\hat{n} = \sum_k b_k^+ \, b_k \qquad (30)$$

In the quasiparticle vacuum

$$\langle \phi_o | \hat{n} | \phi_o \rangle = n = 2 \sum_k V_k^2 \qquad (31)$$

Since the transformation from particle to quasi-particle formalism does not leave the number operator

diagonal, we must add to the Hamiltonian (27) a term $\mu \hat{n}$ and then choose μ such that the expectation value of $\hat{n}$ is equal to the desired number of particles, i.e., we work with the Hamiltonian $(H-\mu\hat{n})$.

Using (29) and (30) we have

$$H - \mu\hat{n} = \sum_k \epsilon_k b_k^+ b_k - G \sum_{kk'} b_k^+, b_{\bar{k}}^+, b_k b_{\bar{k}} \quad (32)$$

where $\epsilon_k = E_k^o - \mu$

i.e. we measure the energies of the particles w.r.t. μ (the chemical potential). Later we choose μ such that $\langle\hat{n}\rangle = n$.

After neglecting the terms V_k^4 we have

$$\langle\phi_o|H-\mu\hat{n}|\phi_o\rangle = 2 \sum_k \epsilon_k V_k^2 - G \left(\sum_k U_k V_k \right)^2 \quad (33)$$

Our task is to minimize (33):

$$\frac{\partial}{\partial V_k} \langle\phi_o|H-\mu\hat{n}|\phi_o\rangle = 0 \quad (34)$$

under the condition

$$U_k^2 + V_k^2 = 1 \quad (35)$$

Using (33) in (34) we have

$$4 \epsilon_k V_k - 2G \left(\sum_k U_k V_k \right) \left(U_k + V_k \frac{\partial U_k}{\partial V_k} \right) = 0$$

On differentiating eq.(35) with respect to V_k we get

$$\frac{\partial U_k}{\partial V_k} = - \frac{V_k}{U_k}$$

therefore we have the set of equations

$$2\epsilon_k V_k - \Delta\left(U_k - \frac{V_k^2}{U_k}\right) = 0 \tag{36}$$

$$U_k^2 + V_k^2 = 1 \tag{37}$$

where we have used

$$\Delta = G \sum_k U_k V_k \tag{38}$$

In view of eq. (37) the set of eqs. (36), (37) can be solved easily. We put

$$V_k = \cos\theta$$

$$U_k = \sin\theta$$

from (36) we have

$$2\epsilon_k U_k V_k = \Delta(U_k^2 - V_k^2)$$

or $$\epsilon_k \sin 2\theta = -\Delta\cos 2\theta$$

or $$\tan 2\theta = -(\Delta/\epsilon_k)$$

$$\therefore \quad \cos 2\theta = U_k^2 - V_k^2 = \frac{\epsilon_k}{\sqrt{\epsilon_k^2 + \Delta^2}}$$

and $$U_k^2 + V_k^2 = 1$$

which together yield

$$U_k^2 = \frac{1}{2}\left(1 + \frac{\epsilon_k}{\sqrt{\epsilon_k^2 + \Delta^2}}\right) \tag{39}$$

$$V_k^2 = \frac{1}{2}\left(1 - \frac{\epsilon_k}{\sqrt{\epsilon_k^2 + \Delta^2}}\right) \tag{40}$$

These are called the Hartree-Fock-Bogoliubov equations.

At this stage we note that had we started with the more general pairing interaction

$$G_{k'k} = \langle k'\bar{k}'|V|k\bar{k}\rangle$$

and carried through the minimization of energy we would have got more general HFB equations;

$$U_k^2 = \frac{1}{2}\left[1 + \frac{E_k^o-\mu}{\sqrt{[(E_k^o-\mu)^2+\Delta_k^2]}}\right] \tag{41}$$

$$V_k^2 = \frac{1}{2}\left[1 - \frac{E_k^o-\mu}{\sqrt{[(E_k^o-\mu)^2+\Delta_k^2]}}\right] \tag{42}$$

$$\Delta_k = \frac{1}{2}\sum_{k'}\frac{\langle k\bar{k}|V|k'\bar{k}'\rangle}{\sqrt{[(E_{k'}^o-\mu)^2+\Delta_{k'}^2]}}\Delta_{k'} \tag{43}$$

also we have

$$n = 2\sum_k V_k^2 \tag{44}$$

The iterative procedure for solving the HFB equations:

We define

$$P(\mu,\Delta_k) = \sum_k \frac{E_k^o}{\sqrt{[(E_k^o-\mu)^2+\Delta_k^2]}} \tag{45}$$

$$Q(\mu,\Delta_k) = \sum_k \frac{1}{\sqrt{[(E_k^o-\mu)^2+\Delta_k^2}} \tag{46}$$

On taking the sum on both sides in eq. (42) and using eq.(44) we have

$$n = N_o - P(\mu,\Delta_k) + \mu\, Q(\mu,\Delta_k)$$

where N_o (=6 for the 2s-1d shell) is the number of the paired states $k\bar{k}$ available. Thus we have

$$\mu = \frac{P(\mu, \Delta_k) - N_o + n}{Q(\mu, \Delta_k)} \tag{47}$$

$$\Delta_k = \frac{1}{2} \sum_{k'} \frac{\langle k\bar{k}|V|k'\bar{k}'\rangle}{\sqrt{[(E^o_{k'} - \mu)^2 + \Delta^2_{k'}]}} \Delta_{k'} \tag{48}$$

The iterative procedure employed in solving the HFB equations is as follows:

a. An intial set of values for μ and Δ_k's is guessed, and also which orbits are occupied.

b. With this set of μ and Δ_k's eq. (47) is iterated to obtain a set of sel-consistent value of μ. In this iteration the Δ_k's are held fixed at the values obtained in the previous step.

c. With the self-consistent value of μ obtained in step b. and the Δ_k's (which were earlier held fixed) we iterate the equation (48) to obtain a self-consistent set of values of Δ_k's. (in this iteration μ remains unchanged).

d. We return to the step b.

2-4 The Technique of Angular Momentum Projection

In the axially symmetric HF, each single-particle state has a given projection quantum number, and hence the HF intrinsic state determinant ϕ_K has a given total projection quantum number K, which is the sum of individual single-particle projection quantum numbers.

It is usual to describe the actual nuclear state with a definite angular momentum J. Therefore the determinant ϕ_K is written as a linear superposition of states ψ_K^J

$$|\phi_K\rangle = \sum_J a_J |\psi_K^J\rangle \tag{49}$$

Operating with the rotational operator

$$\hat{R}(\Omega) = e^{-i\alpha J_z} e^{-i\theta J_y} e^{-i\gamma J_z} \tag{50}$$

where Ω represents the Eulerian angles α,θ and γ we have

$$\hat{R}(\Omega)|\phi_K\rangle = \sum_{JM} a_J D_{MK}^{J*}(\Omega) |\psi_M^J\rangle$$

where D_{MK}^{J*} are the matrix elements of the rotation operator in the basis $|\psi_K^J\rangle$.

Now multiplying both sides by $D_{M'K}^{J'}$ and integrating over Ω, where

$$\int d\Omega \equiv \int_0^{2\pi} d\gamma \int_0^{\pi} d\theta \sin\theta \int_0^{2\pi} d\alpha$$

and using the property

$$\int d\Omega D_{MK}^{J*}(\Omega) D_{M'K}^{J'}(\Omega) = \frac{8\pi^2}{2J+1} \delta_{JJ'} \delta_{MM'}$$

and finally omitting the primes, we have

$$|\psi^J_M\rangle = \frac{2J+1}{8\pi^2 a_J} \int d\Omega\ D^J_{MK}(\Omega)\ \hat{R}(\Omega)\ |\phi_K\rangle \qquad (51)$$

We say, in view of the eq. (51), that the nuclear state of angular momentum J and projection M in the Laboratory frame (belonging to a rotational band) is projected out of the deformed intrinsic HF/HFB state $|\phi_K\rangle$, which has axial symmetry and projection K on the z-axis of the intrinsic frame. Thus the projection operator is written as

$$P^J_{MK} = \frac{2J+1}{8\pi^2 a_J} \int d\Omega\ D^J_{MK}(\Omega)\ \hat{R}(\Omega)$$

Putting M=K the equation

$$|\psi^J_K\rangle = \frac{2J+1}{8\pi^2 a_J} \int d\Omega\ D^J_{KK}(\Omega)\ \hat{R}(\Omega)\ |\phi_K\rangle$$

can be represented by

$$|\psi^J_K\rangle = P^J_{KK}\ |\phi_K\rangle \qquad (52)$$

where

$$P^J_{KK} = \frac{2J+1}{8\pi^2 a_J} \int d\Omega\ D^J_{KK}(\Omega)\ \hat{R}(\Omega) \qquad (53)$$

The projection operator P^J_{KK} has the following properties

(a) $(P^J_{KK})^+ = P^J_{KK}$

(b) $(P^J_{KK})^2 = P^J_{KK}$

(c) $[H, P^J_{KK}] = 0$

The property (a) follows from the fact that angular momentum operator is hermitian. Again, the projection operator operating on $|\psi_K^J\rangle$ gives back $(\psi_K^J\rangle$; therefore property (b) follows. Property (c) is obvious as H is a scalar operator and cannot change the angular momentum of a state on which it operates.

In view of the above properties of the projection operator and eq (52), the energy of the projected state $|\psi_K^J\rangle$

$$E_J = \frac{\langle\psi_K^J \mid H \mid \psi_K^J\rangle}{\langle\psi_K^J \mid \psi_K^J\rangle}$$

reduces to

$$E_J = \frac{\langle\phi_K|H\, P_{KK}^J|\, \phi_K\rangle}{\langle\phi_K|P_J|\phi_K\rangle} \qquad (54)$$

Using the expression for P_{KK}^J and $\hat{R}\,(\Omega)$ from (53) and (50) respectively and using the fact that $\exp(-i\gamma J_z)$ and $\exp(-i\alpha J_z)$ Operating on the bras and kets $\langle\phi_K|$ and $|\phi_K\rangle$ produces $\exp(-i\gamma K)$ and $\exp(-i\alpha K)$ respectively and using

$$D_{KK}^J(\Omega) = e^{-i\alpha K}\, d_{KK}^J(\Theta)\, e^{-i\gamma K}$$

and cancelling the integrations over α and γ we finally have

$$E_J = \frac{\int_0^\pi d\Theta \sin\Theta\, d_{KK}^J(\Theta)\, \langle\phi_K|H\, e^{-i\Theta J_y}\, |\phi_K\rangle}{\int_0^\pi d\Theta \sin\Theta\, d_{KK}^J(\Theta)\, \langle\phi_K|e^{-i\Theta J_y}|\phi_K\rangle} \qquad (55)$$

2-5 Calculation of Subshell Occupation Numbers in Terms of States with Good Angular Momentum Projected from the HFB Intrinsic States (The Projected Hartree-Fock-Bogoliubov Method)[4,5]

The axially symmetric intrinsic deformed HFB states with K=0 can be written as

$$|\phi_0\rangle = \prod_{im} (U_{im} + V_{im}\ b^+_{im}\ b^+_{i\bar{m}})\,|0\rangle \qquad (56)$$

where the creation operators b^+_{im} can be expressed as

$$b^+_{im} = \sum_j C^j_{im}\ a^+_{jm}$$

$$b^+_{i\bar{m}} = \sum_j (-)^{j-m} C^j_{im}\ a^+_{j,-m}$$

Here, the index i is used to distinguish between different states with the same m, and j labels the spherical single-particle orbitals $1d_{5/2}$, $2s_{1/2}$, and $1d_{3/2}$. The wave function in eq.(56) can be reduced into the form

$$|\phi_0\rangle = N \exp\left[\frac{1}{2} \sum_{\alpha\beta} f_{\alpha\beta}\ a^+_\alpha\ a^+_\beta\right] |0\rangle \qquad (57)$$

with

$$f_{\alpha\beta} = \sum_i C^{j_\alpha}_{im_\alpha}\ C^{j_\beta}_{im_\beta}\ \frac{V_{im_\alpha}}{U_{im_\alpha}}\ \delta_{m_\alpha,-m_\beta} \qquad (58)$$

Here α denotes the quantum numbers $(j_\alpha m_\alpha)$ and N is the normalization constant.

Now, the subshell occupation number operator is given by

$$\hat{\eta}_j = \sum_{m=-j}^{+j} a^+_{jm} \; a^+_{jm} \tag{59}$$

The expectation value of this operator with respect to the angular momentum projected HFB states

$$|\psi^J_o\rangle = P^J_{oo} \; |\phi_o\rangle$$

can be written, in view of the eq. (55), as

$$\eta_j = \frac{\int_o^\pi p(\theta) \; d^J_{oo}(\theta) \; \sin\theta \; d\theta}{\int_o^\pi n(\theta) \; d^J_{oo}(\theta) \; \sin\theta \; d\theta} \tag{60}$$

where $p(\theta) = \langle\phi_o| \eta_j \; e^{-i\theta J_y} \; |\phi_o\rangle$ (61)

and $n(\theta) = \langle\phi_o| \; e^{-i\theta J_y} |\; \phi_o\rangle$ (62)

they are related as

$$p(\theta) = n(\theta) \; [\sum_{m=-j}^{+j} \rho_{jm,jm}] \tag{63}$$

The density matrix $\rho(\theta)$ is defined as

$$\rho(\theta) = \frac{M(\theta)}{1+M(\theta)} \tag{64}$$

where $M(\theta) = F(\theta) \; f^+$ (65)

and $F_{\alpha\beta}(\theta) = \sum_{m'_\alpha m'_\beta} d^{j_\alpha}_{m_\alpha m'_\alpha}(\theta) \; d^{j_\beta}_{m_\beta m'_\beta}(\theta) \; f_{j_\alpha m'_\alpha, \; j_\beta m_\beta}$ (66)

Again the normalizations n^J are given by

$$n^J = \int_0^\pi n(\theta)\, d^J_{oo}(\theta)\, \sin\theta\, d\theta \qquad (67)$$

where $$n(\theta) = \sqrt{\det[1+M(\theta)]} \qquad (68)$$

It is worth mentioning here that an approximate expression for the subshell occupation numbers can easily be obtained in terms of the expectation value of the operator η_j with respect to the intrinsic state $|\phi_o\rangle$,

$$\eta_j \quad \langle\phi_o|\hat{\eta}_j|\,\phi_o\rangle = \sum_{im} |C^j_{im}|^2\, V^2_{im}$$

The above equation provides the values for occupation numbers averaged over the complete rotational band. These band averaged values are likely to differ significantly from the estimates based on the use of the projected J=O wave functions.

2-6 Effective Interactions:

In this thesis we shall discuss the results obtained with seven alternative two-body interactions. Five of these were obtained via least square search so as to optimize the agreement between the calculated and the observed properties of nuclei in the 2s-1d shell. In what follows we give first a short hand level for each Hamiltonian, then the number of parameters which were adjusted via least square search, and then a brief description concerning the origin of the interaction[6].

K+SPE (no free parameters) This is the realistic effective Hamiltonian derived from the Hamada-Johnston potential for nucleon-nucleon scattering by Kuo[7].

KB+SPE (no free parameters) This isthe realistic effective Hamiltonian derived by Kuo and Brown[8] in the same manner as the preceeding interaction.

K+12FP (9 free parameters) The interaction[6] involves 9 two-body matrix elements which were parametrized. All other two body matrix elements were held fixed at the Kuo values.

RIP (11 free parameters) The interaction[6] stands for '' Radial Integral Parametrization''. It involves 11 linearly independent combination of radial Talmi integrals which were considered free parameters.

MSDI (4 free parameters) The letters MSDI[6] stand for '' Modified Surface Delta Interaction''. The four parameters are the two surface-delta strengths (one for T=C and one for T=1 interaction), plus two modifying monopole strengths (one for T=0 and one for T=1).

PW (18 free parameters) This interaction was obtained by Preedom and Wildenthal[9]. Those two-body metrix elements of K+SPE which do not involve the $d_{3/2}$ orbit were varied as free parameters. In addition, the centroids of the $d_{5/2}$-$d_{5/2}$ and $d_{3/2}$-$d_{3/2}$ interaction were also varied.

W (63 free parameters) This interaction was obtained by Wildenthal[10]. The 63 two-body matrix elements characterizing the full 2s-1d shell interaction were obtained by requiring a stable best fit to a set of 440 data which sampled every 2s-1d shell system with experimentally known energies. An overall constraint was the assumption that the matrix elements decreased with A as $(18/A)^{0.3}$.

In all the above interactions the single-particle energies were also considered as free parameters.

2-7 Calculational Procedure

The initial guess for the wave functions involved in HF/HFB iterations was generated by diagonalising the Nilsson Hamiltonian

$$h = h_o + \alpha \vec{l}.\vec{s} + \gamma \vec{l}.\vec{l} + \delta_o Q_o^2 \qquad (69)$$

in the basis of the 2s-1d shell states.

The expression for the matrix elements of the Q_o^2 operator is as follows

$$\langle n'l'j'm' | Q_o^2 | nljm \rangle$$

$$= \langle n'l'|r^2|nl\rangle \begin{bmatrix} j & 2 & j' \\ \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} j & 2 & j' \\ m & 0 & m' \end{bmatrix} \qquad (70)$$

The Calculation of Subshell Occupation Numbers

The subshell occupation numbers for protons and neutrons were calculated as follows. The results of the HFB calculations are summarized in terms of the expansion coefficients C_{im}^j and the amplitudes (U_{im}, V_{im}) appearing in eq (56).

These values were used to first set up the 12X12 matrix $f_{\alpha\beta}$ defined by eq (58). We next calculate the matrices $F_{\alpha\beta}(\theta)$, $M(\theta)$ and $\rho(\theta)$ defined by, respectively, the eqns (66), (65) and (64). These calculations were carried out for 20 values of the Gaussian quadrature points lying in the range $(0, \pi/2)$. Finally the subshell occupation numbers were calculated by using the eqs (60) and (63).

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CHAPTER 3

RESULTS AND DISCUSSION

3-1 A Comparative Study of the PHFB and the Shell-Model Approaches Employing the Wildenthal Interaction

Here we present the estimate for subshell occupation numbers for a number of 2s-1d shell nuclei in the frame work of the shell model as well as the PHFB approaches, employing the Wildenthal interaction. The calculations are aimed at assessing the relative efficacy of the PHFB method vis-a-vis the calculation of occupation numbers. We have shown our results in Fig. 3 and Tables I and II. The results given here provide a first-ever explicit demonstration of the quantitative reliability of the PHFB method. The PHFB results are throughout in excellent agreement with the shell model estimates; the maximum discrepancy occurs in the case of the nucleus ^{32}S where the PHFB method overestimates the shell model values by only about 7.7%.

We have also given in Fig.3 the experimental values for the subshell occupation numbers for neutrons resulting from various stripping and pickup reactions indicated in Table I. The error bars associated with the experimental numbers displayed in Fig. 3 summarize the differences in the predictions resulting from various reactions. It is seen

that the Wildenthal interaction in conjunction with the shell model frame work provides a reasonably good overall description of the available experimental data. A noticeable descrepancy however occurs in the case of the nucleus ^{28}Si where the Wildenthal interaction overestimates (underestimates) the subshell occupation numbers for the neutrons in the $1d_{3/2}(1d_{5/2})$ orbits. This is not however surprising in view of the fact that the structure of this nucleus involves maximum number of valance configurations for protons and neutrons in the 2s-1d shell.

3-2 A Comparative Study of the PHFB Predictions for Proton and Neutron Subshell Occupation Number Resulting from the Various Effective Interactions Operating in the 2s-1d Shell.

In Fig. 4 and Table II we have presented the subshell occupation numbers resulting from various effective interactions described earlier. One notices widely different predictions resulting from different interactions for nuclei heavier than ^{24}Mg. The interaction lebelled as K,KS, K+12FP and RIP fail to reproduce the observed $(d_{5/2})^n$ occupation numbers in the nuclei ^{26}Mg, ^{28}Si, ^{30}Si, and ^{32}S. As far as the interactions MSDI, PW, and W are concerned they give nearly identical results agreeing reasonably well with the experiment throughout the shell for the $(d_{5/2})^n$ orbit. Whereas the MSDI interaction leads to extremely

poor predictions for the $(d_{3/2})^n$ orbit in the case of the nuclei $^{28,30}Si$ and ^{32}S, the interaction P'/ leads to poor results only in the case of ^{32}S.

In Table III we have compared the subshell occupation numbers resulting from the PHFB as well as the spectral method[1] for some N=Z nuclei in the 2S-1d shell. It is seen that the predictions resulting from the two methods are quite different. The differences are not entirely unanticipated since the spectral method is not expected to provide an accurate description of the properties associated with the ground states which are usually more than five widths away from the centroid of the energy distribution assumed to be Gaussian.

TABLE I. Ground-state subshell occupation numbers from experimental data.[1]

Nucleus	Protons				Neutrons				Reaction	Ref.
	$j=\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	$l=2$	$j=\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	$l=2$		
^{20}Ne	0.5	0.3	1.1						(d,T)	2
					0.2	0.5	1.3		(T,α)	3
					0.1	0.8	1.0		(T,α)[a]	4
					0.2	0.8	1.0		(T,α)[b]	4
					0.3			1.7	(d,p)	5
^{22}Ne	0.6	0.1	1.3						(d,T)	2
	0.5			1.5					(d,n)	6
	0.5			1.5					(T,d)	7
					0.3			3.7	(d,p)	5
^{24}Mg	0.4	0.3	3.3						(d,T)	9
	0.3	0.7	3.0						(d,T)	8
	0.3	0.6	3.1						(d,T)	10
					0.3	0.6	3.1		(d,t)	10
					0.2	0.4	3.4		(p,d)	11

Contd...

Contd... TABLE I

Nucleus	Protons j= 1/2	3/2	5/2	l=2	Neutrons j= 1/2	3/2	5/2	l=2	Reactions	Ref.
^{26}Mg	0.5	0.1	3.4						(d,T)	9
	0.3	0.4	3.3						(d,T)[a]	12
	0.6	0.5	2.9						(d,T)[b]	12
	0.6	0.1	3.3						(T,d)	13
	0.4	0.4	3.2						(T,d)	14
	0.4	0.0	3.6						(T,d)	15
					0.4	0.5	5.2		(d,t)	16
					0.4	0.6	5.0		(T,α)	16
					0.3	0.6	5.1		(T,α)	17
^{28}Si	0.6	1.1	4.3						(d,T)	18
	0.3	0.8	4.7						(d,T)	8
					0.6	0.3	5.1		(p,d)	11
					0.5			5.5	(T,α)	19
					1.1			4.9	(d,p)	20

Contd.....

Contd... TABLE I

Nucleus	Protons				Neutrons				Reactions	Ref.
	$j=\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	l=2	$j=\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	l=2		
^{30}Si	0.4			5.6					(t,α)	22
	0.6	0.1	5.3						(T,α)	23
	0.3	0.4	5.3						(T,d)	24
	0.2	0.4	5.5						(T,d)	25
	0.5	0.1	5.4						(T,d)	15
					1.0	1.2	5.8		(p,d)	21
					1.1	1.3	5.6		(d,t)	26
					0.7			7.3	(T,α)	27
					0.8	1.6	5.6		(d,p)	25

Contd....

Contd... TABLE I

Nucleus	Protons				Neutrons				Reactions	Ref.
	$j=\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	l=2	$j=\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	l=2		
^{32}S	1.4	0.6	6.0						(d,T)	28
	1.0	1.2	5.8						(T,d)	29
					1.1	1.0	5.8		(p,d)	11
					1.2	1.0	5.9		(T,α)	30
					1.1	0.7	5.8		(d,p)	13
^{36}Ar	1.8	2.3	5.9						(d,n)	31
					1.8	2.3	5.8		(d,p)	20
					1.7	2.6	5.7		(d,p)	32

a,b) Distorted Wave Born Approximation analysis with two sets of parameters.

TABLE II. The subshell occupation numbers for protons and neutrons resulting from various effective interactions are tabulated. The intrinsic quadrupole moments have been given in units of b^2, where b is the oscillator parameter.

Nucleus	Interactions	$\langle Q_0^2 \rangle$	Protons			Neutrons		
			$j=\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	$j=\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$
^{20}Ne	K+SPE	15.18	0.53	0.26	1.30	0.53	0.26	1.30
	KB+SPE	15.21	0.60	0.23	1.30	0.60	0.23	1.30
	K+12FP	15.56	0.67	0.27	1.15	0.67	0.27	1.15
	RIP	15.30	0.92	0.15	1.01	0.92	0.15	1.01
	MSDI	14.73	0.58	0.12	1.38	0.58	0.12	1.38
	PW	15.30	0.60	0.23	1.26	0.60	0.23	1.26
	W	15.36	0.68	0.20	1.20	0.68	0.20	1.20
	W(SM)		(0.51)	(0.28)	(1.21)	(0.51)	(0.28)	(1.21)
^{24}Mg	K+SPE	18.94	0.49	0.56	2.94	0.49	0.56	2.94
	KB+SPE	18.68	0.50	0.43	3.07	0.50	0.43	3.07
	K+12FP	19.01	0.42	0.66	2.90	0.42	0.66	2.90
	RIP	18.98	0.68	0.37	2.99	0.68	0.37	2.99
	MSDI	17.69	0.50	0.18	3.36	0.50	0.18	3.36
	PW	17.61	0.26	0.28	3.49	0.26	0.28	3.49
	W	18.03	0.33	0.33	3.38	0.33	0.33	3.38
	W(SM)		(0.45)	(0.56)	(2.99)	(0.45)	(0.56)	(2.99)
^{26}Mg	K+SPE	20.98	0.45	0.38	3.21	0.60	1.77	3.63
	KB+SPE	-18.28	0.98	0.37	2.52	1.47	0.78	3.75
	K+12FP	21.03	0.38	0.47	3.18	0.43	1.96	3.61
	RIP	-18.31	1.48	0.25	2.21	1.68	0.85	3.47
	MSDI	-15.33	0.76	0.12	2.98	0.95	0.20	4.85
	PW	-15.71	0.34	0.27	3.26	0.64	0.39	4.97
	W	-15.99	0.43	0.27	3.12	0.73	0.39	4.88
	W(SM)		(0.35)	(0.45)	(3.20)	(0.56)	(0.62)	(4.82)

Contd.....

Contd... TABLE II

Nucleus	Interactions	$\langle Q_o^2 \rangle$	Protons $j=\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	Neutrons $j=\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$
^{28}Si	K+SPE	-22.77	1.33	1.12	3.55	1.33	1.12	3.55
	KB+SPE	-22.24	1.45	0.97	3.58	1.45	0.97	3.58
	K+12FP	-23.01	1.26	1.25	3.49	1.26	1.25	3.49
	RIP	-22.37	1.65	1.09	3.25	1.65	1.09	3.25
	MSDI	-18.85	1.12	0.37	4.51	1.12	0.37	4.51
	PW	-19.82	0.90	0.61	4.49	0.90	0.61	4.49
	W	-20.28	0.99	0.65	4.36	0.99	0.65	4.36
	W(SM)		(0.70)	(0.68)	(4.62)	(0.70)	(0.68)	(4.62)
^{30}Si	K+SPE	-20.01	1.29	0.36	3.85	1.41	1.87	4.78
	KB+SPE	-18.65	1.45	0.66	3.88	1.64	1.55	4.89
	K+12FP	-20.46	1.13	1.01	3.86	1.26	2.15	4.76
	RIP	-19.78	1.71	0.93	3.35	1.67	1.82	4.55
	MSDI	6.96	1.05	0.11	5.02	1.95	0.07	5.94
	PW	9.76	0.10	0.06	5.83	0.58	1.45	5.93
	W	-15.69	0.59	0.24	5.17	1.22	1.29	5.48
	W(SM)		(0.68)	(0.58)	(4.74)	(1.41)	(1.44)	(5.15)
^{32}S	K+SPE	-17.52	1.36	1.70	4.98	1.36	1.70	4.98
	KB+SPE	-13.62	1.78	1.00	5.27	1.78	1.00	5.27
	K+12FP	15.03	0.23	2.14	5.63	0.23	2.14	5.63
	RIP	-16.91	1.77	1.61	4.69	1.77	1.61	4.69
	MSDI	0.43	1.98	0.04	5.94	1.98	0.04	5.94
	PW	0.08	1.98	0.04	5.94	1.98	0.04	5.94
	W	13.21	1.14	0.96	5.86	1.14	0.96	5.86
	W(SM)		(1.41)	(1.16)	(5.42)	(1.41)	(1.16)	(5.42)

Contd....

Contd.... TABLE II

Nucleus	Interactions	$\langle Q_o^2 \rangle$	Protons $j=\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	Neutrons $j=\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$
^{34}S	K+SPE	-14.75	1.32	1.40	5.23	1.57	2.61	5.75
	KB+SPE	-10.01	1.88	0.42	5.67	1.94	2.12	5.86
	K+12FP	-14.54	0.47	1.96	5.52	0.68	3.48	5.77
	RIP	-12.73	1.88	1.15	5.04	1.89	2.14	5.89
	MSDI	1.21	1.98	0.04	5.94	1.97	2.08	5.94
	PW	-5.85	1.97	0.05	5.93	1.98	2.01	5.94
	W	-11.86	1.46	0.70	5.77	1.66	2.38	5.88
	W(SM)		(1.66)	(0.73)	(5.61)	(1.76)	(2.48)	(5.76)
^{36}Ar	K+SPE	-14.37	1.60	2.64	5.68	1.60	2.64	5.68
	KB+SPE	-11.46	1.90	2.19	5.82	1.90	2.19	5.82
	K+12FP	-15.34	0.93	3.31	5.68	0.93	3.31	5.68
	RIP	-10.16	1.93	2.08	5.91	1.93	2.08	5.91
	MSDI	-11.65	1.84	2.18	5.90	1.84	2.18	5.90
	PW	-9.52	1.94	2.05	5.93	1.94	2.05	5.93
	W	-13.52	1.62	2.45	5.84	1.62	2.45	5.84
	W(SM)		(1.78)	(2.68)	(5.54)	(1.78)	(2.68)	(5.54)

The numbers given in brackets denote the exact shell model results obtained with the Wildenthal(W) interaction.

TABLE III. Comparison of the subshell occupation numbers resulting from the PHFB as well as the spectral method for some N=Z nuclei in 2s-1d shell.

Nucleus	Interaction	Spectral Method $j=\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	PHFB Method $j=\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$
^{20}Ne	PW	0.5	0.1	3.4	1.2	0.5	2.5
	K+12FP	0.6	0.2	3.1	1.3	0.5	2.3
	RIP	1.7	0.0	2.2	1.8	0.3	2.0
	MSDI	0.5	0.0	3.5	1.2	0.2	2.8
^{24}Mg	PW	0.8	0.5	6.7	0.5	0.6	7.0
	K+12FP	0.9	1.2	5.9	0.8	1.3	5.8
	RIP	3.2	0.4	4.5	1.4	0.7	6.0
	MSDI	1.5	0.1	6.4	1.0	0.4	6.7
^{28}Si	PW	1.4	1.1	9.5	1.8	1.2	9.0
	K+12FP	1.4	2.3	8.3	2.5	2.5	7.0
	RIP	3.6	1.2	7.2	3.3	2.2	6.5
	MSDI	2.3	0.5	9.2	2.2	0.7	9.0
^{32}S	PW	2.7	2.0	11.3	4.0	0.1	11.9
	K+12FP	1.8	3.8	10.4	0.5	4.3	11.3
	RIP	3.9	2.4	9.7	3.5	3.2	9.4
	MSDI	3.3	1.4	11.2	3.9	0.1	11.9
^{36}Ar	PW	3.8	4.2	12.0	3.9	4.1	11.9
	K+12FP	2.3	6.0	11.7	1.9	6.6	11.4
	RIP	4.0	4.3	11.7	3.9	4.2	11.8
	MSDI	3.9	4.1	12.0	3.7	4.4	11.8

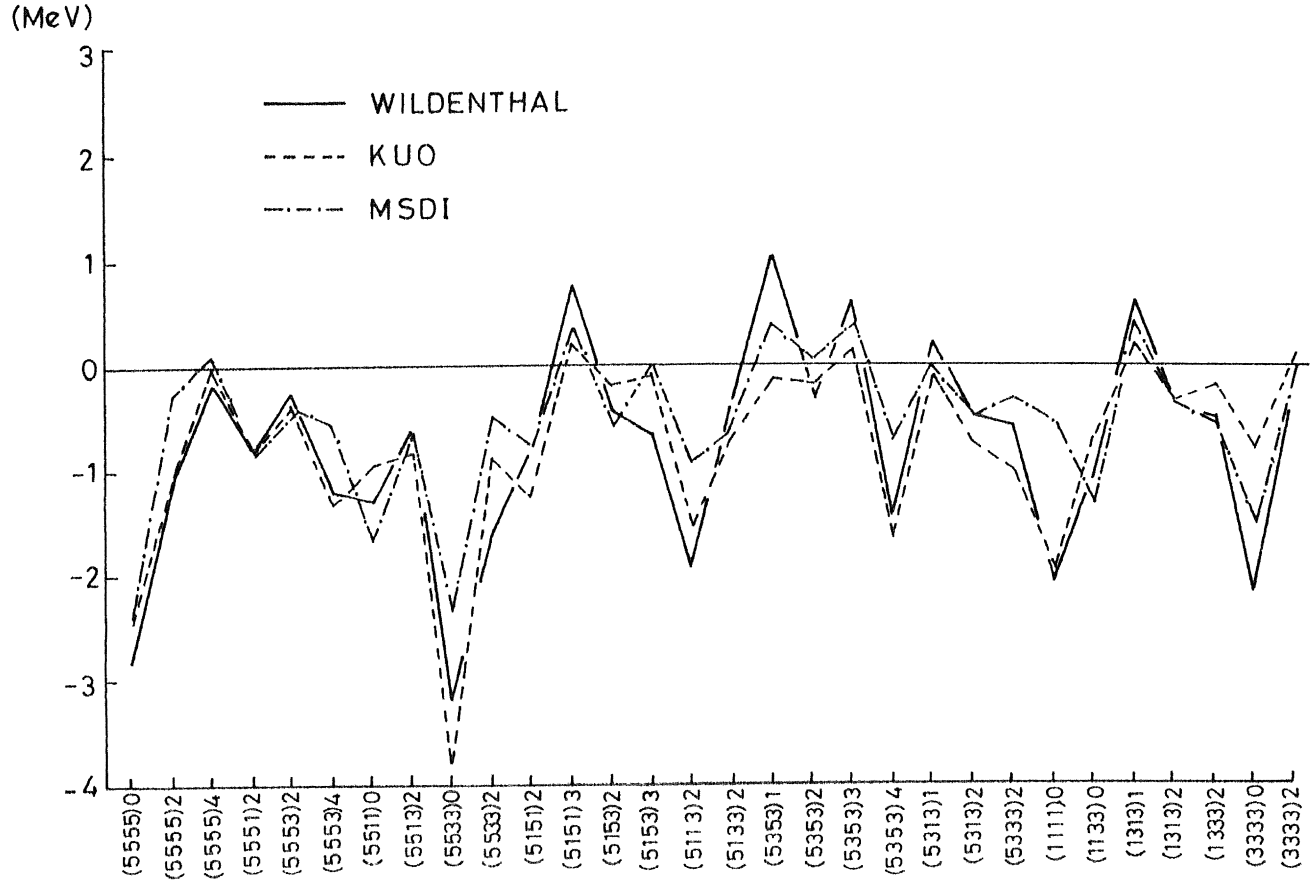

FIG.1 T=1 MATRIX ELEMENTS OF VARIOUS EFFECTIVE INTERACTIONS

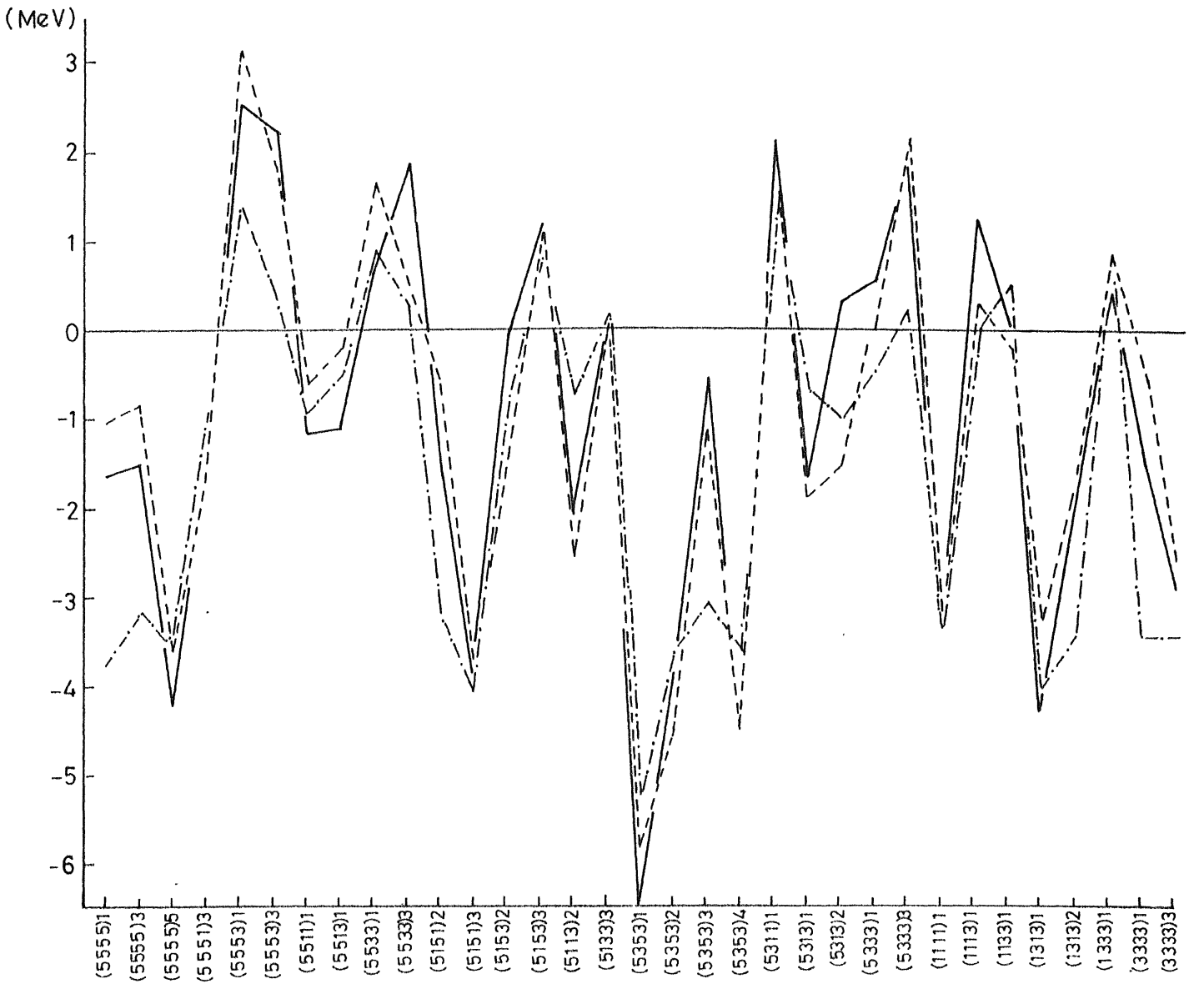

FIG. 2 T=0 MATRIX ELEMENTS

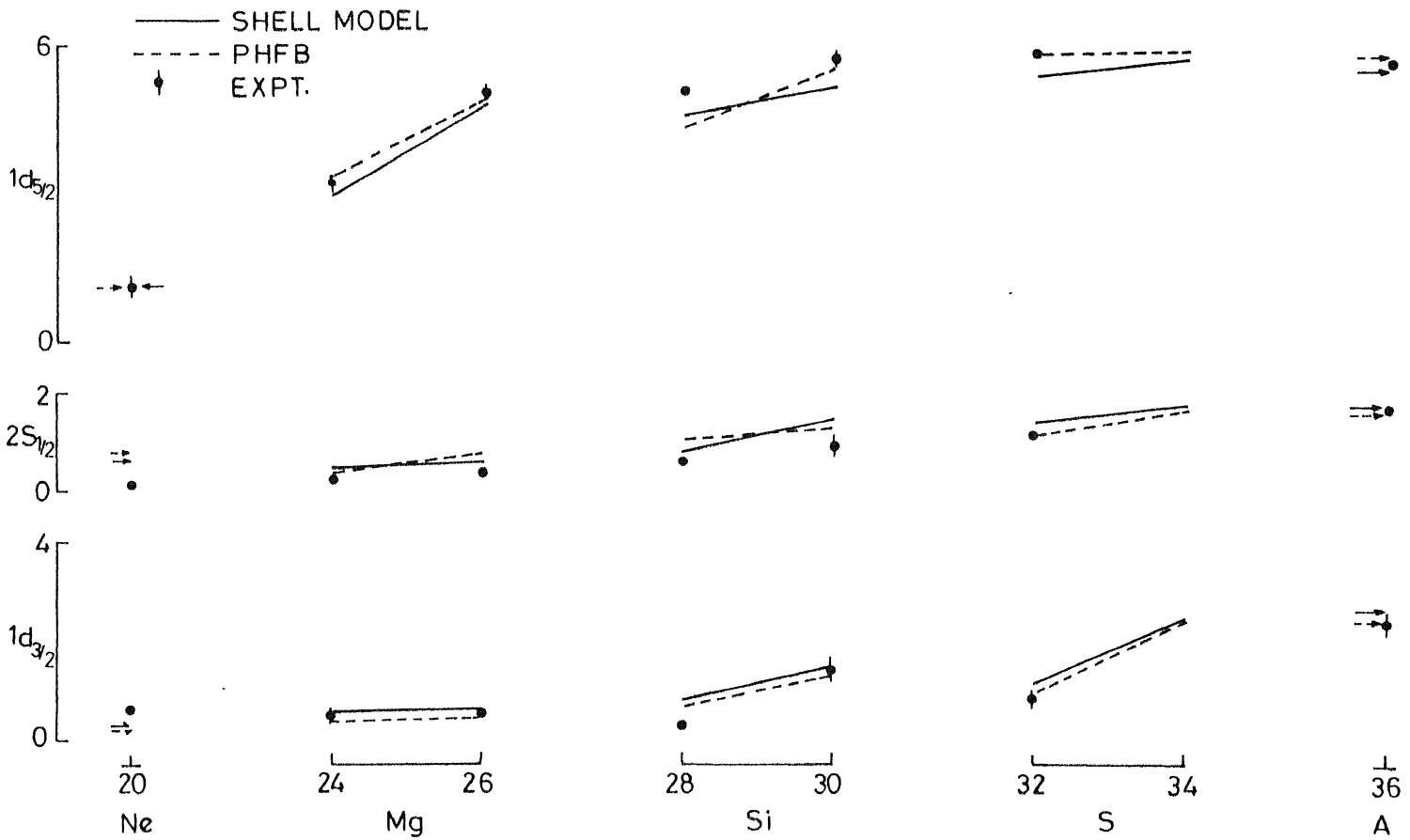

FIG. 3 COMPARISON OF THE RESULTS OBTAINED IN THE SHELL MODEL AND THE PHFB FRAMEWORK

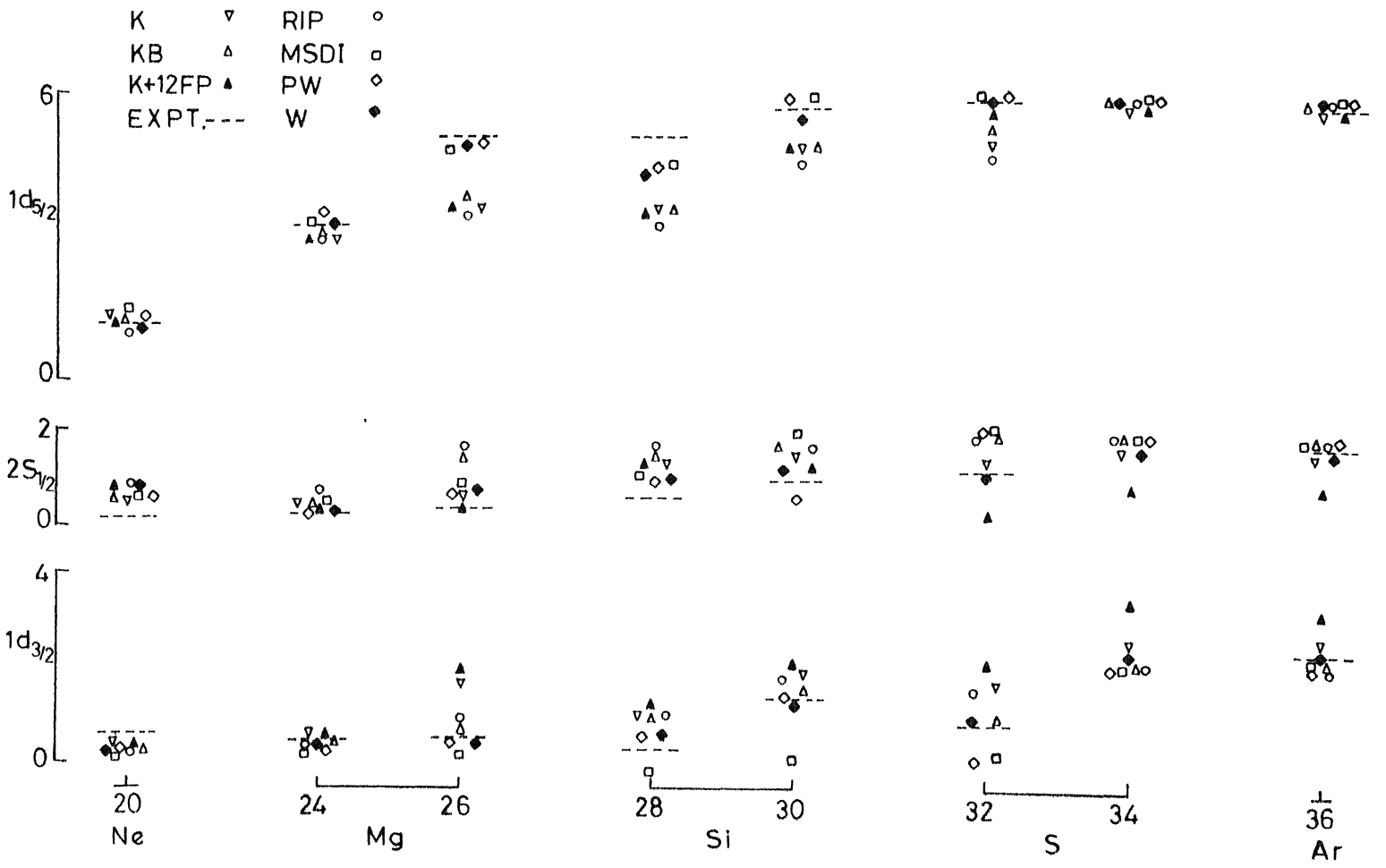

FIG.4 SUBSHELL OCCUPATION NUMBERS FOR NEUTRONS VIA THE PHFB METHOD

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